



# Locally corrected Nyström method for EM scattering by bodies of revolution

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## Abstract

The Nyström method allows for high order solutions to integral equations. Modifying the Nyström method with local corrections for singular integrands allows the method to be applied to the inherently singular kernels of frequency-domain electromagnetic integral equations. Here the efficiency and high order convergence properties of the Nyström method are brought together with the body of revolution (BOR) geometry to create an efficient approach to solving this important class of problems. In this paper we summarize the locally corrected Nyström method and the magnetic field integral equation (MFIE) for a BOR scatterer. Results are presented that demonstrate high order convergence and excellent agreement with reference data.

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## 1. Introduction

Frequency domain integral equation methods for scattering problems have traditionally used projection methods to find an approximate solution for the tangential magnetic fields on the surface of perfectly conducting scatterers. Projection methods involve selecting a finite set of basis functions, defining an inner product, and choosing a set of testing functions. If done properly, this yields an  $N$ -by- $N$  matrix equation, where  $N$  is the dimension of the basis set. In addition,  $N$  is proportional to the electrical area of the scatterer (that is, the area divided by a square wavelength), so when the scatterer is large or the wavelength is small,  $N$  can become very large. Direct methods of solving  $N$ -by- $N$  matrix equations incur a computational cost of

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$O(N^3)$ ; iterative methods incur a cost of  $O(N^2)$  per iteration. So-called “fast” iterative methods can drive the latter cost down to  $O(N \log N)$  per iteration [21]. It is clear that techniques that significantly decrease  $N$  are very desirable.

Due to the high computing power required by integral equation methods for scattering problems, it is of significant practical value to develop techniques that handle simpler geometries, such as bodies of revolution (BOR). When the scatterer possesses axial symmetry, projection methods admit a solution based on a Fourier series decomposition of the tangential magnetic field in the azimuthal dimension, [11]. The Fourier series is truncated after  $N_f$  terms where  $N_f = O(\rho_{\max}/\lambda)$ ,  $\lambda$  is the wavelength, and  $\rho_{\max}$  is the maximum girth of the scatterer about its symmetry axis. This results in a block matrix in which the off-diagonal blocks are identically zero, leading to a decoupling of the diagonal blocks. The dimension of each diagonal block is  $N/N_f$ . For instance, if  $N = 1000$  and  $N_f = 10$ , then the BOR projection algorithm requires the solution of 10 matrices each having dimension 100. Compare this to the conventional projection algorithm that demands the solution of a single matrix having dimension 1000. When  $N$  exceeds  $10^5$ , the computational savings are tremendous. In addition, BORs provide a valuable class of validation problems for advanced general-purpose scattering algorithms. They are frequently used as calibration standards in scattering measurement facilities. It is known that a BOR algorithm can analyze the scattering from such scatterers at higher frequency and with higher accuracy than any other available method. Consequently the problem of computing the scattering characteristics of BORs have been studied by many authors. Among the integral equation based methods that exploit azimuthal symmetry, the frequency-domain BOR Method of Moments (BOR-MoM) is perhaps the most mature. The mathematical formulation was first given in [1], while the first BOR-MoM algorithm was described in [11,12]. A MoM formulation was used in [8] for a perfectly electrically conducting (PEC) body of revolution coated with layered, homogeneous dielectric. Two of the widely used BOR-MoM FORTRAN codes for EM radiation and scattering analysis are JRMBOR [20] and CICERO/CARLOS-BOR [14]. Other methods include PDE based techniques for inhomogeneous BOR problems, [9,16–18], where scalar potentials are introduced to reduce the number of unknowns for large scatterers, hybrid method [15], pseudospectral method [25], and FFT method [7].

It is known that projection methods exhibit poor convergence characteristics in the vicinity of geometry-induced field singularities, such as edges and tips, because of the use of finite bases to approximate infinite functions. The simplicity and speed advantages of Nyström method make it a natural alternative (to projection methods) for high order solutions to integral equations of scattering problems. This has not been done widely due to the limitations of the conventional Nyström method to regular kernels. However, this limitation can be overcome by “singularity extraction,” or, more efficiently, “local corrections [22].” This method of integrating singular kernels was used to solve electromagnetic boundary integral equations in [4]. It was further extended to volume integral equations and to dealing with geometric singularities by incorporating Gauss–Jacobi polynomials in [6]. The purpose of this paper is to combine locally corrected Nyström method with the advantages of BOR geometry. The result is a BOR solver, AFITBOR.

The following section describes the locally corrected Nyström method. A BOR coordinate system is defined in Section 3 and the magnetic field integral equation (MFIE) is discretized. More details on local corrections and BOR savings are also discussed. In Section 4, we present numerical results for some canonical scatterers that demonstrate the high order convergence of the method. The paper is concluded in Section 5.

## 2. Locally corrected Nyström method

### 2.1. Conventional Nyström method

Consider the following simple integral equation for the sake of reviewing the locally corrected Nyström method:

$$g(x) = \int_a^b G(x, x')u(x') dx'$$

First, the interval  $[a, b]$  is divided into  $N_s$  subintervals and a  $N_a$ -point quadrature rule approximates the integration on each subinterval to arrive at the following:

$$g(x) \approx \sum_{p=1}^{N_s} \sum_{q=0}^{N_a-1} \omega_q G(x, x_q^p)u(x_q^p),$$

where  $x_q^p$  is the  $q$ th abscissa on the  $p$ th subinterval. In practice, the quadrature rule need not be identical on each subinterval, but we shall impose this requirement for notational simplicity and clarity. Evaluating  $g(x)$  at the  $n$ th abscissa of the  $m$ th subinterval, i.e., each abscissa from the underlying quadrature rule, gives

$$g(x_n^m) \approx \sum_{p=1}^{N_s} \sum_{q=0}^{N_a-1} \omega_q G(x_n^m, x_q^p)u(x_q^p). \tag{2.1}$$

At this point, the approximate solutions to the integral equations come from simply solving a system of equations with unknowns  $u_q^p = u(x_q^p)$ . However, an obvious problem for electromagnetic integral equations is the singularity of the kernel which makes evaluation at  $x_n^m = x_p^q$  impossible. In addition, quadrature convergence is slow when  $\|x_n^m - x_p^q\| \ll \lambda$ .

The method of local corrections allows extending the Nyström method to singular kernels. Local correction schemes entail replacing the weights and abscissas of the conventional Nyström relation of Eq. (2.1) with “locally corrected” ones when  $\|x_n^m - x_p^q\| < \delta$  for some chosen separation threshold  $\delta$ . The locally corrected weights incorporate the singular behavior of the kernel, while avoiding actual evaluation at a singular abscissa point. The separation threshold ensures that the actual kernel evaluations occur far enough away from the singularity to allow the Nyström quadrature rule to achieve high-order convergence. The nature of the local corrections is explored in the following section.

### 2.2. Local corrections

The idea of local corrections is explained here in the context of a one-dimensional problem. The BORs are two-dimensional surfaces, but it will be shown how the BOR problem can be reduced to several one-dimensional integral equations. Define a new quadrature rule on this subinterval which incorporates the singularity into the quadrature weights

$$\int_a^b G(x, x')u(x') dx' \approx \sum_{n=0}^{N_a-1} u_n \bar{\omega}_n.$$

We require the quadrature rule to be exact for polynomials of degree less than or equal to  $N_a - 1$ . Let  $P_n(x)$  be polynomials of degree  $n$  on the subinterval; typically the Legendre polynomials are used. The new weights  $\bar{\omega}_n$  are found by solving the following system of equations.

$$\begin{aligned} \bar{\omega}_1 P_0(x_1) + \dots + \bar{\omega}_{N_a-1} P_0(x_{N_a}) &= \int_a^b G(x, x') P_0(x') dx', \\ \bar{\omega}_1 P_1(x_1) + \dots + \bar{\omega}_{N_a-1} P_1(x_{N_a}) &= \int_a^b G(x, x') P_1(x') dx', \\ &\dots\dots\dots \\ \bar{\omega}_1 P_{N_a-1}(x_1) + \dots + \bar{\omega}_{N_a-1} P_{N_a-1}(x_{N_a}) &= \int_a^b G(x, x') P_{N_a-1}(x') dx'. \end{aligned}$$

The new quadrature rule can be applied without evaluation of the kernel at a singular point. Note the right hand side of the system of equations for the corrected weights  $\bar{\omega}_n$  must be calculated carefully using a strong numerical quadrature due to the singular behavior of the kernel (e.g. Duffy transform, adaptive quadrature). For small problems, the computation of the corrected weights is the most time consuming portion of the solution process. Fortunately, the computation of locally corrected weights is only done once at every point. Therefore, the local correction process is an  $O(N)$  computation, where, again,  $N$  is the number of unknowns.

Once the computation of the corrected weights is complete, assembling the linear system involves only evaluation of the kernel at the abscissa points. Also, in the case of electromagnetic scatterers, if the direction of the incident field changes this only affects the right hand side of the integral equation. Hence, the weights can be reused since the kernel is unaffected by the direction of the incident field. As for changes in frequency of the incident field which will affect the kernel, the corrected weights can be reused if the local corrections are computed prudently. The kernel can be written as the product of a frequency dependent part and a frequency independent part. The frequency independent part contains the singularity, so the corrected weights can be calculated on the frequency independent portion and modified by simple multiplication against the frequency dependent part to account for the given frequency. This allows computing the corrected weights once and using them over a range of frequencies on the same geometry.

### 3. MFIE for a BOR scatterer

The magnetic field integral equation (MFIE) over a PEC BOR geometry can be reduced to a one-dimensional problem along the curve defining the BOR. A Fourier series expansion in the azimuthal direction of the BOR allows this reduction which is done in the following manner.

Given an incident field  $\mathbf{H}^i(\mathbf{r})$  we wish to solve the MFIE [23] for a surface current  $\mathbf{J}_s$ ,

$$\hat{\mathbf{n}} \times \mathbf{H}^i(\mathbf{r}) = \frac{1}{2} \mathbf{J}_s(\mathbf{r}) - \hat{\mathbf{n}} \times \int_S \mathbf{J}_s(\mathbf{r}') \times \nabla' g(\mathbf{r}, \mathbf{r}') ds', \quad \mathbf{r} \in S, \quad (3.1)$$

where  $g(\mathbf{r}, \mathbf{r}') = \exp(ik|\mathbf{r} - \mathbf{r}'|)/[4\pi|\mathbf{r} - \mathbf{r}'|]$  is the free space Green function for the Helmholtz equation and  $\hat{\mathbf{n}}$  is the outward pointing unit vector on the surface  $S$ .

The BOR geometry is created by rotating a curve  $(\rho, z)$  about the  $z$ -axis, as shown in Fig. 1. The curve is parameterized by its arc-length  $\ell \in [0, \bar{\ell}]$ , where  $\bar{\ell}$  is the total length of the generating arc. In the BOR geometry, the surface current will have two vector components, one in the azimuthal direction and the other in the direction of the defining arc, that is,  $\mathbf{J}_s = \hat{\ell} J_\ell + \hat{\phi} J_\phi$ . The unit vectors in these directions are given by

$$\begin{aligned} \hat{\ell} &= \rho_*(\ell) \cos(\phi) \hat{\mathbf{x}} + \rho_*(\ell) \sin(\phi) \hat{\mathbf{y}} + z_*(\ell) \hat{\mathbf{z}}, \\ \hat{\phi} &= -\sin(\phi) \hat{\mathbf{x}} + \cos(\phi) \hat{\mathbf{y}}, \end{aligned}$$

where  $\rho_*(\ell) = \rho(\ell)/N(\ell)$ ,  $z_*(\ell) = z(\ell)/N(\ell)$  and  $N(\ell) = \sqrt{[\rho'(\ell)]^2 + [z'(\ell)]^2}$ . (The prime indicates differentiation with respect to  $\ell$ .) The BOR coordinate system parameterizes the surface into two variables  $\ell$  and  $\phi$ ,

$$\begin{aligned} x(\ell, \phi) &= \rho(\ell) \cos(\phi), \\ y(\ell, \phi) &= \rho(\ell) \sin(\phi), \\ z(\ell, \phi) &= z(\ell). \end{aligned}$$

The periodicity of the BOR geometry in the azimuthal direction allows the solution to be expanded into a Fourier series in the  $\phi$ -direction hence reducing the integral equation to only  $\ell$  dependence. Of course, this

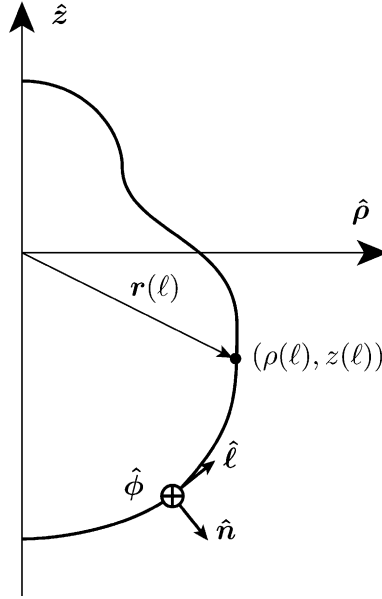


Fig. 1. The BOR geometry. The generating arc is parameterized by arc length  $l$ . Unit vectors are:  $\hat{l}$  in the direction of increasing  $l$ ,  $\hat{\phi} \doteq \hat{z} \times \hat{\rho}$ , and the outward normal  $\hat{n} = \hat{\phi} \times \hat{l}$ .

will yield a separate equation for each mode of the expansion, but each will involve fewer unknowns than if the BOR were treated as a two-dimensional problem.

Using the above description of the BOR geometry and some change of variables, the vector integral equation (3.1) can be written as the following two scalar integral equations [13]

$$\frac{J_\ell(\ell, \phi)}{2} = \int_0^{2\pi} \int_0^{\bar{\ell}} \{ \alpha_1(\ell, \ell_1, \phi_1) J_\ell(\ell_1, \phi_1 + \phi) + \alpha_2(\ell, \ell_1, \phi_1) J_\phi(\ell_1, \phi_1 + \phi) \} d\ell_1 d\phi_1 + \hat{\ell} \cdot (\hat{n} \times \mathbf{H}^i),$$

$$\frac{J_\phi(\ell, \phi)}{2} = \int_0^{2\pi} \int_0^{\bar{\ell}} \{ \alpha_3(\ell, \ell_1, \phi_1) J_\ell(\ell_1, \phi_1 + \phi) + \alpha_4(\ell, \ell_1, \phi_1) J_\phi(\ell_1, \phi_1 + \phi) \} d\ell_1 d\phi_1 + \hat{\phi} \cdot (\hat{n} \times \mathbf{H}^i).$$

The kernel functions  $\alpha_{1,\dots,4}$  are given by

$$\alpha_1(\ell, \ell_1, \phi_1) = -T(\ell_1)G(\ell, \ell_1, \phi_1) \left\{ ([\rho(\ell_1) - \rho(\ell)]z_*(\ell_1) - [z(\ell_1) - z(\ell)]\rho_*(\ell_1)) \cos(\phi_1) - 2\rho(\ell)z_*(\ell_1) \sin^2\left(\frac{\phi_1}{2}\right) \right\},$$

$$\alpha_2(\ell, \ell_1, \phi_1) = -T(\ell_1)G(\ell, \ell_1, \phi_1)[z(\ell_1) - z(\ell)] \sin(\phi_1),$$

$$\alpha_3(\ell, \ell_1, \phi_1) = -T(\ell_1)G(\ell, \ell_1, \phi_1) \{ \rho(\ell_1)\rho_*(\ell)z_*(\ell_1) - \rho(\ell)\rho_*(\ell_1)z_*(\ell)[z(\ell_1) - z(\ell)]\rho_*(\ell)\rho_*(\ell_1) \} \sin(\phi_1),$$

$$\alpha_4(\ell, \ell_1, \phi_1) = -T(\ell_1)G(\ell, \ell_1, \phi_1) \left\{ ([\rho(\ell_1) - \rho(\ell)]z_*(\ell) - [z(\ell_1) - z(\ell)]\rho_*(\ell)) \cos(\phi_1) - 2\rho(\ell_1)z_*(\ell) \sin^2\left(\frac{\phi_1}{2}\right) \right\},$$

where

$$T(\ell_1) = \rho(\ell_1) \sqrt{[\rho'(\ell_1)]^2 + [z'(\ell_1)]^2},$$

$$G(\ell, \ell_1, \phi_1) = \frac{(1 + ikR)e^{ikR}}{4\pi R^3},$$

$$R = \sqrt{[\rho(\ell) - \rho(\ell_1)]^2 + [z(\ell) - z(\ell_1)]^2 + 4\rho(\ell)\rho(\ell_1) \sin^2\left(\frac{\phi_1}{2}\right)}.$$

Both  $J_\ell$  and  $J_\phi$  are periodic in the azimuthal direction and thus both have a Fourier series expansion in the  $\phi$ -variable.

$$J_\ell(\ell, \phi) = \sum_{n=-\infty}^{\infty} j_n^\ell(\ell) e^{in\phi},$$

$$J_\phi(\ell, \phi) = \sum_{n=-\infty}^{\infty} j_n^\phi(\ell) e^{in\phi}.$$

Note that each coefficient is a function of  $\ell$  but is free of dependence on  $\phi$ . Our focus now will be on an integral equation for each of the individual coefficient functions  $j_n^\ell(\ell)$  and  $j_n^\phi(\ell)$ . The incident field can also be expressed as a Fourier series on the surface of the BOR

$$[\hat{\ell} \cdot (\hat{\mathbf{n}} \times \mathbf{H}^i)]_m = \int_0^{2\pi} [\hat{\ell} \cdot (\hat{\mathbf{n}} \times \mathbf{H}^i)] e^{-im\phi} d\phi,$$

$$[\hat{\phi} \cdot (\hat{\mathbf{n}} \times \mathbf{H}^i)]_m = \int_0^{2\pi} [\hat{\phi} \cdot (\hat{\mathbf{n}} \times \mathbf{H}^i)] e^{-im\phi} d\phi.$$

Using orthogonality of the exponentials we isolate the unknown coefficient functions

$$\frac{j_m^\ell(\ell)}{2} = \int_0^{2\pi} \int_0^{\bar{\ell}} [\alpha_1(\ell, \ell_1, \phi_1) j_m^\ell(\ell_1) + \alpha_2(\ell, \ell_1, \phi_1) j_m^\phi(\ell_1)] d\ell_1 d\phi_1 + [\hat{\ell} \cdot (\hat{\mathbf{n}} \times \mathbf{H}^i)]_m,$$

$$\frac{j_m^\phi(\ell)}{2} = \int_0^{2\pi} \int_0^{\bar{\ell}} [\alpha_3(\ell, \ell_1, \phi_1) j_m^\ell(\ell_1) + \alpha_4(\ell, \ell_1, \phi_1) j_m^\phi(\ell_1)] d\ell_1 d\phi_1 + [\hat{\phi} \cdot (\hat{\mathbf{n}} \times \mathbf{H}^i)]_m.$$

Since  $j_m^\ell(\ell)$  and  $j_m^\phi(\ell)$  have no  $\phi_1$  dependence we can write the integral equation as

$$\frac{j_m^\ell(\ell)}{2} = \int_0^{\bar{\ell}} [j_m^\ell(\ell_1) G_m^1(\ell, \ell_1) + j_m^\phi(\ell_1) G_m^2(\ell, \ell_1)] d\ell_1 + [\hat{\ell} \cdot (\hat{\mathbf{n}} \times \mathbf{H}^i)]_m,$$

$$\frac{j_m^\phi(\ell)}{2} = \int_0^{\bar{\ell}} [j_m^\ell(\ell_1) G_m^3(\ell, \ell_1) + j_m^\phi(\ell_1) G_m^4(\ell, \ell_1)] d\ell_1 + [\hat{\phi} \cdot (\hat{\mathbf{n}} \times \mathbf{H}^i)]_m,$$

where

$$G_m^i(\ell, \ell_1) = \int_0^{2\pi} \alpha_i(\ell, \ell_1, \phi_1) d\phi_1. \quad (3.2)$$

Thus, the original integral equation (3.1) is reduced to a one-dimensional problem. However, the one-dimensional problem is only solving for one component of the Fourier series solution with  $N_f$  Fourier modes; the problem must be solved  $N_f$  times. The value of  $k$  will determine the number of terms of the Fourier series that must be computed to achieve adequate accuracy.

### 3.1. BOR local corrections

Since the BOR geometry allows the integral equation (3.1) to be recast as a one-dimensional problem, the local corrections are done just as indicated in Section 2.2 using the kernel  $G_m^i(\ell, \ell_1)$  given in (3.2). However, evaluation of this kernel involves integration which can be problematic near the singular point. Assume the singular point is in a subinterval  $(a_i, b_i)$ . To evaluate an integral of the form

$$I(\ell) = \int_{a_i}^{b_i} P_n(\ell_1) G_m^i(\ell, \ell_1) d\ell_1$$

as would be necessary for local corrections is done as follows. Treat the integral as a double integral

$$I(\ell) = \int_{a_i}^{b_i} \int_0^{2\pi} P_n(\ell_1) \alpha_i(\ell, \ell_1, \phi_1) d\phi_1 d\ell_1$$

and split the region of integration into two parts

$$I(\ell) = \int_{a_i}^{b_i} \int_{-\epsilon}^{\epsilon} P_n(\ell_1) \alpha_i(\ell, \ell_1, \phi_1) d\phi_1 d\ell_1 + \int_{a_i}^{b_i} \int_{\epsilon}^{2\pi-\epsilon} P_n(\ell_1) \alpha_i(\ell, \ell_1, \phi_1) d\phi_1 d\ell_1.$$

Now the integral over the region  $[\epsilon, 2\pi - \epsilon] \times [a_i, b_i]$  has no singularity and can thus be handled with standard quadrature. (Exploiting symmetry of the integrand, we can recast the integral in  $\phi$  to one over the region  $[\epsilon, \pi]$ , but no further analytical simplifications are known.) On the other hand, evaluating the integral over  $[-\epsilon, \epsilon] \times [a_i, b_i]$  does contain the singularity and must be treated with care to get accurate results. It is evaluation of this integral where adaptive quadrature or the Duffy transform would be employed [5,10,24,26]. When choosing  $\epsilon$ , there are two forces at work. First, it is of interest to make  $\epsilon$  as small as possible in order to limit the extent of the local correction domain. On the other hand, if  $\epsilon$  is too small, then the use of standard quadrature rules in the  $[\epsilon, 2\pi - \epsilon] \times [a_i, b_i]$  region may give poor results from having the endpoints too close to the singularity. A reasonable compromise for choosing  $\epsilon$  is to attempt to minimize the aspect ratio of the actual region which  $[-\epsilon, \epsilon] \times [a_i, b_i]$  represents on the BOR geometry. In any other subinterval  $[a_j, b_j]$  which does not contain the singular point, evaluations of  $G_m^i(\ell, \ell_1)$  can be done with standard quadrature rules.

### 3.2. BOR savings

A BOR surface discretized into  $N_\ell$  points in the longitudinal direction and  $N_\phi$  points in the azimuthal direction would have  $4N_\ell N_\phi$  unknowns. Taking advantage of the periodicity in the azimuthal direction, the surface current can be decomposed into a Fourier series in  $\phi$ . Hence, the solution can be found for each term of the series. If  $N_f$  Fourier terms are included, then  $N_f$  systems of  $2N_\ell$  unknowns must be solved as opposed to one system with  $4N_\ell N_\phi$  unknowns. Thus the solution is found by inverting  $N_f$  matrices with size  $2N_\ell$  as opposed to inverting one  $(4N_\ell N_\phi) \times (4N_\ell N_\phi)$  matrix.

## 4. Numerical results

### 4.1. Convergence

The locally corrected Nyström method optimally yields convergence of  $h^p$  where  $h$  is the size of the subintervals, and  $p$  is the order of the quadrature rule. For the canonical example of the sphere, using the Mie series allows the approximations from the Nyström method to be compared to a very close approximation of the exact solution [3]. The error should decrease exponentially for the  $p$ -refinement. As for  $h$ -refinement, the error should decrease linearly on a log–log scale. We test the the locally corrected Nyström results with two perfectly conducting geometries, a sphere and an oblate spheroid. In both cases, a uniform plane wave is incident upon the scatterer, propagating in the  $-\hat{z}$  direction.

In the case of  $p$ -refinement, we use a PEC sphere with  $ka = 12$ . The BOR in this case is generated by a semicircle. The solution process divides the semicircle into 5 subintervals and progressively raises the degree of the polynomial from degree zero to degree six. After degree two, the linear behavior on the semi-log scale becomes apparent. The results are shown in Fig. 2.

For the  $h$ -refinement, the same geometry and parameters are used, but we begin with a fixed polynomial degree, and proceed to increase the number of subintervals which of course reduces the size of  $h$ . In Fig. 3, we see the linear behavior on the log–log scale as expected, and the slope is much steeper when the polynomial degree is  $p = 4$  than when the degree is  $p = 2$ .

### 4.2. Sphere

We compare the surface currents and far-zone scattered fields produced by AFITBOR to the actual Mie series solution [3]. Fig. 4(a) and(b) show very good agreement between the radar cross-section (RCS) results obtained from the Mie series and AFITBOR for a unit-radius conducting sphere. We compute results for two frequencies,  $k = 4.7$  and  $k = 10$  ( $k$  is the wavenumber). Surface current comparisons are shown in Fig. 5(a) and (b); again, the Mie series and AFITBOR solutions are in good agreement.

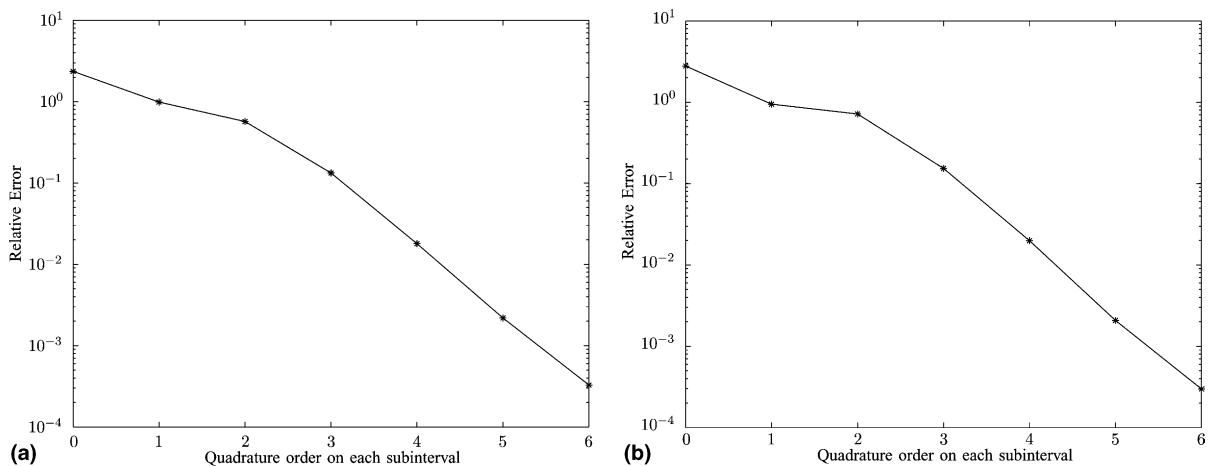


Fig. 2. Exponential convergence for  $p$ -refinement for a unit-radius conducting sphere with  $k = 12$ . (a) Convergence of  $J_l$ . (b) Convergence of  $J_\phi$ .



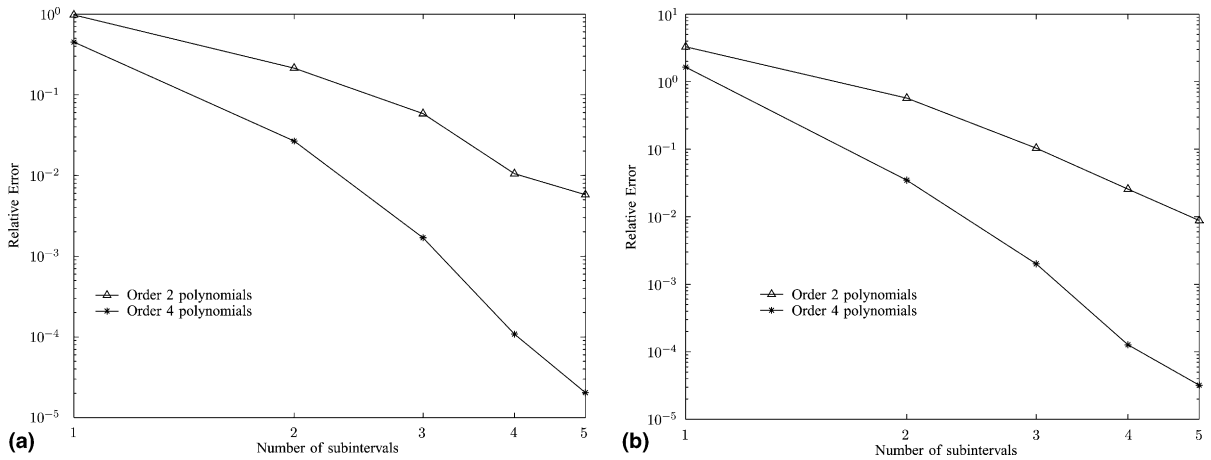


Fig. 3. Geometric convergence for  $h$ -refinement for a unit-radius conducting sphere with  $k = 12$ . (a) Convergence of  $J_t$ . (b) Convergence of  $J_\phi$ .

### 4.3. Oblate spheroid

Here we apply the method to a non-spherical BOR for which an analytic solution is not readily available. The only change in the AFITBOR code is to modify the definition of the BOR defining curve. The oblate spheroid is the BOR defined by rotating half an ellipse with major axis  $a = 2$  and minor axis  $b = 1$  about its minor axis. The RCS for both the  $\phi\phi$  and  $\theta\theta$  polarizations is plotted in Fig. 6(a) and (b) for  $ka = 4.7$ . The bistatic RCS results compare well with those from a moment-method code, AIM [2], which uses a triangular facet representation of the scatterer surface and the Rao–Wilton–Glisson basis functions [19]. The surface currents from AFITBOR are plotted in Fig. 7(a) and (b).

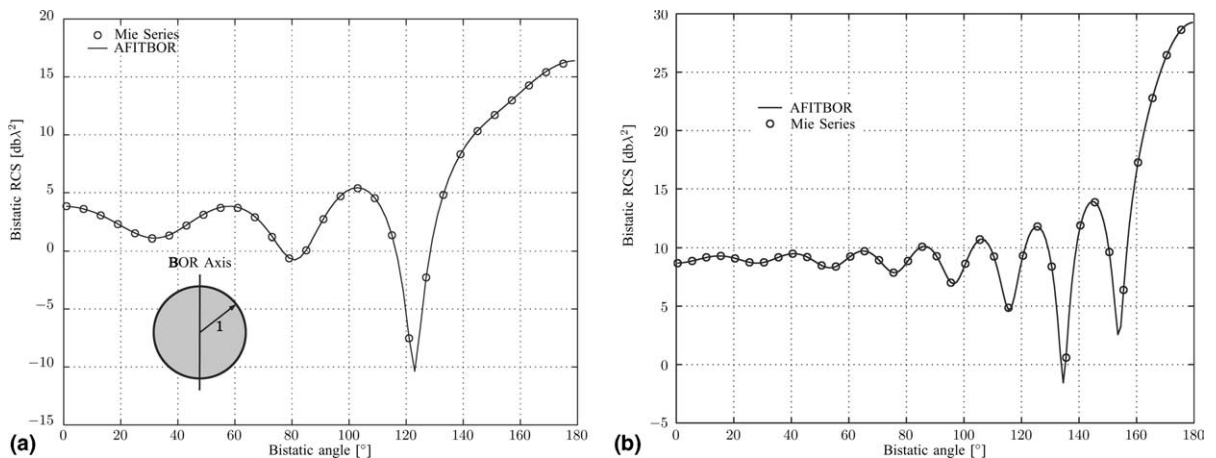


Fig. 4. Bistatic RCS of a unit-radius conducting sphere at two frequencies. Incident and scattered fields are both  $\theta$  polarized. (a)  $k = 4.7$ . (b)  $k = 10$ .

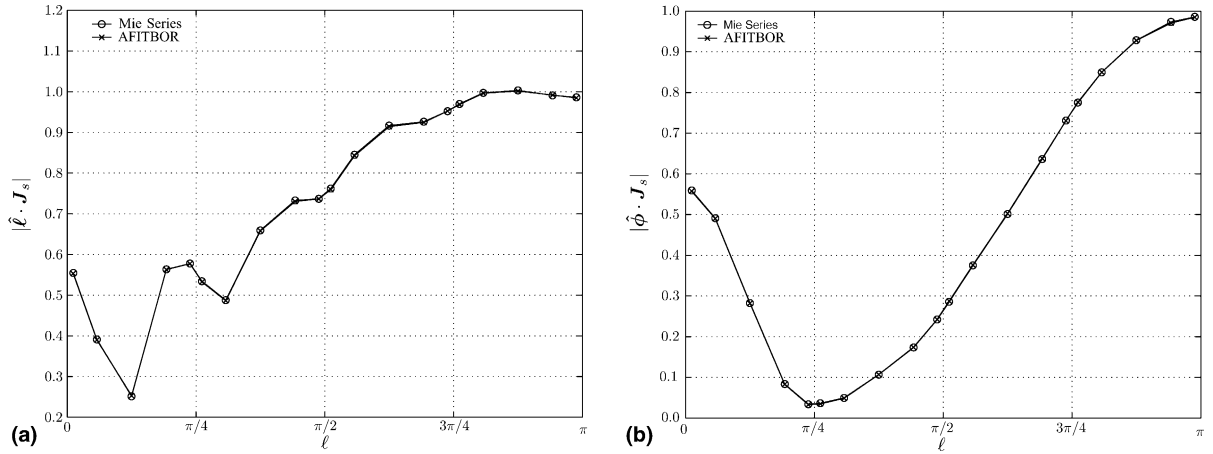


Fig. 5. Current density magnitude of the unit-radius conducting sphere with  $ka = 4.7$ . Incident field is  $\theta$  polarized. (a)  $\hat{l}$ -directed current density. (b)  $\hat{\phi}$ -directed current density.

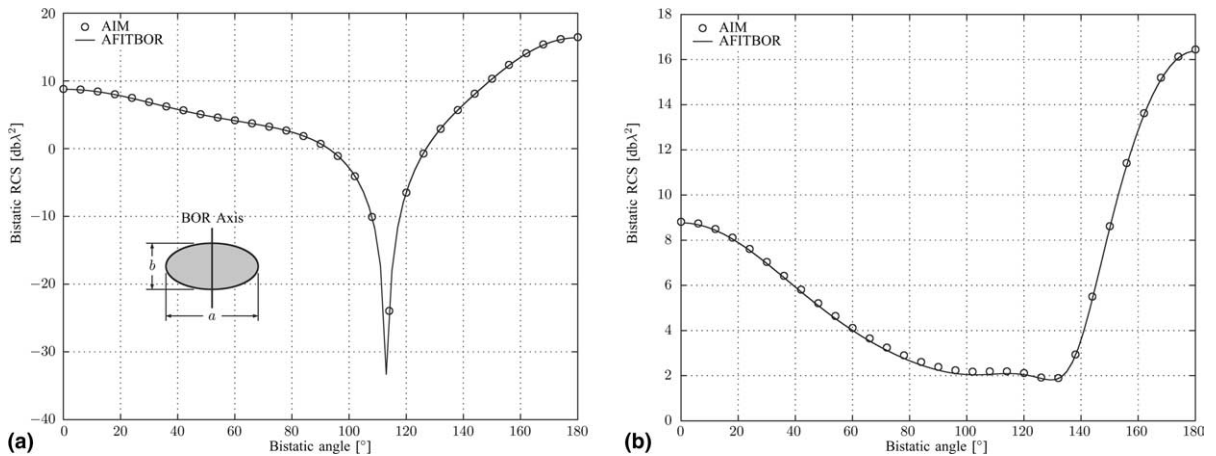


Fig. 6. Bistatic RCS for an oblate spheroid with  $ka = 4.7$ . The oblate spheroid major axis is  $a = 2$  and its minor axis is  $b = 1$ . (a)  $\theta\theta$ -pol. (b)  $\phi\phi$ -pol.

## 5. Conclusion

The purpose of this paper was to demonstrate another application of the locally corrected Nyström method. By combining the advantages of the BOR geometry with the simplicity of the Nyström approach, we offer an alternative method for computing fields scattered by these special geometries. The advantages of the AFITBOR approach are simplicity and strong convergence properties. The bulk of the computational work is isolated in the computation of the corrected weights which is time-consuming due to the repeated integration of singular kernels. However, once the corrected weights are tabulated on a given geometry, the direction and frequency of the incident field can be changed and the corrected weights need not be re-computed. The only computation beyond the corrected weights is a matrix fill which entails only function evaluation and the matrix inversion. Therefore, the most advantageous situation would be solving for many

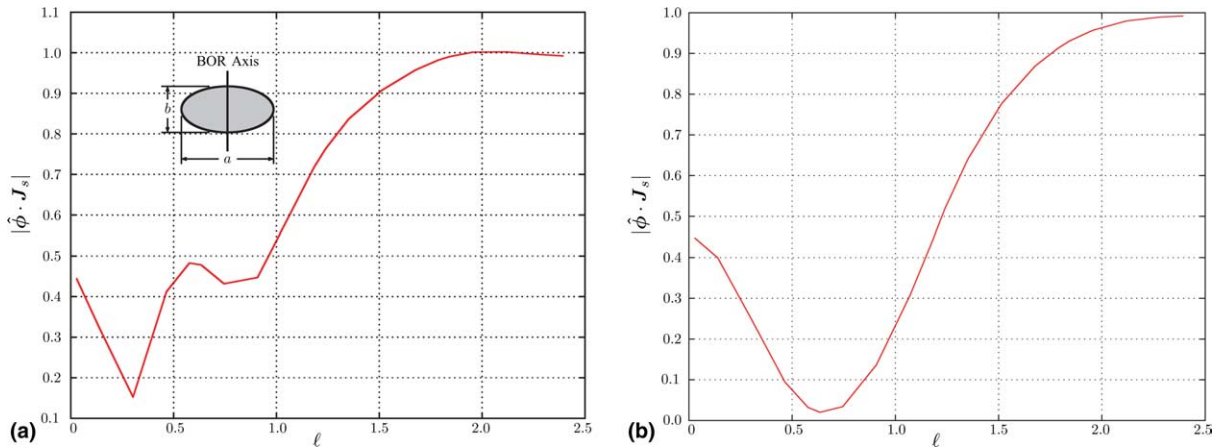


Fig. 7. Current density magnitude computed by AFITBOR of the conducting oblate spheroid with  $ka = 4.7$ . Incident field is  $\theta$  polarized. (a)  $\hat{\ell}$ -directed current density. (b)  $\hat{\phi}$ -directed current density.

different frequencies or angles of incidence on the same geometry. The first run would be time consuming, but each subsequent run would be very efficient.

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